Stochastic programming models in the energy sector

Andy Philpott
Electric Power Optimization Centre
University of Auckland
www.epoc.org.nz

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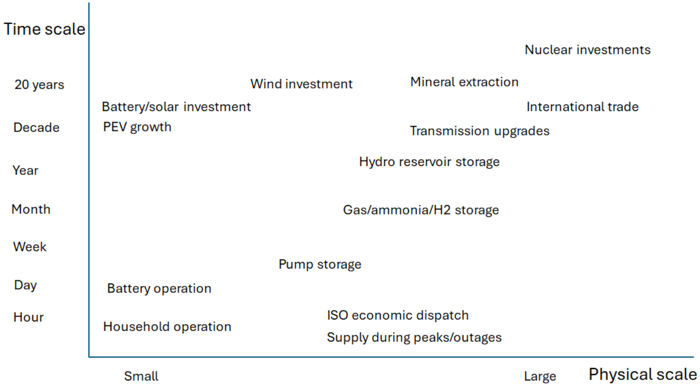
Optimization models widely used in the energy sector

- Decision makers are engineers;
- Physical constraints that must be satisfied;
- Uncertainties determined by physical sciences (geology, meteorology);
- Large investments financed by debt: risk minimization;
- Energy includes heat, transport, electricity,...;
- Focus of this tutorial: electricity;
- Papavasiliou, A., 2024. Optimization models in electricity markets. Cambridge University Press.

Problems vary in time scale and physical scale



Range of energy optimization problems vary in time scale and physical scale.

The green energy transition

- There are many optimization challenges arising from the transition from fossil fuels to renewable energy.
- E.J. Anderson, M.C. Ferris, A.B. Philpott, M. Anitescu, P. Cramton, S. Geng, R. Green, T. Homem de Mello, O. Huber, V. Leclere and R. Sioshansi (2025). Ten challenges for mathematical modeling of the energy transition, downloadable from www.epoc.org.nz.
- M.C. Ferris and A.B. Philpott (2025). Optimizing green energy sytems, Informs Journal of Optimization (online), downloadable from www.epoc.org.nz.
- Will energy markets get us there?

View stochastic programming through a market lens

- Most electricity systems are now run as competitive markets.
  - Electricity producers maximize profits.
  - Electricity consumers maximize welfare from purchases.
- Competitive markets are supposed to deliver social optimum.
  - When social planning problem is single-period, convex, deterministic. . .
    . . . Lagrangian gives prices that match market outcome with social plan.
- What happens when the social plan is a stochastic optimization?

What we will NOT focus on in this tutorial

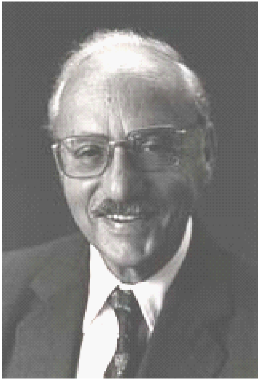
- Particular optimization techniques (e.g., mixed-integer programming models for unit commitment);
- Optimization models of specific technologies (e.g., hydrogen production using electrolysis);
- Fuel exploration and production (e.g., geothermal well drilling);
- Electricity transmission/distribution planning;
- Stochastic optimization in "exchange-type" markets;
- Imperfect competition and game theory;
- Reliability/chance constraints;
- What's left?

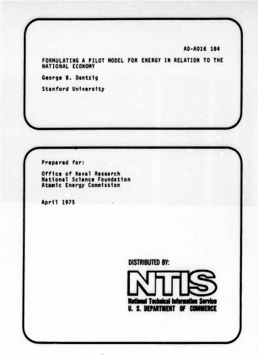
This tutorial in three parts

1. Short-term models (hours/days)
   - social plan minimizing cost
   - maximizing profit given prices
2. Multistage and medium-term models (weeks/months)
   - social plan minimizing cost
   - maximizing profit given prices
3. Long-term models (years/decades)
   - social plan minimizing cost
   - maximizing profit given prices

Planning Investment Level Over Time

[Planning, Investment, Labor, Operations, and Technology]





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FORMULATING A PILOT MODEL FOR ENERGY IN RELATION TO THE NATIONAL ECONOMY

George B. Dantzig

Stanford University

Prepared for:

Office of Naval Research
National Science Foundation
Atomic Energy Commission

April 1975

DISTRIBUTED BY:

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National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

PILOT mode created at Stanford after 1973 oil crisis (US Government wanted zero oil imports by 1980).

Planning Investment Level Over Time

"In spite of the fact that the PILOT model is the real McCoy - a powerful tool for making policy decisions - decision makers do not line up to use PILOT or, for that matter, any other model."

[D. J. Albers, C. Reid and G.B. Dantzig, An Interview with George B. Dantzig: The Father of Linear Programming, The College Mathematics Journal , Sep., 1986, Vol. 17, No. 4 (Sep., 1986), 292-314.]

One way to make PILOT more useful?

- Focus on determining a first-stage decision . . .
- . . . enable some recourse in later decisions.
- Leads naturally to a two-stage model with scenarios.
- The first-stage decision is less sensitive to future variability.
- The first-stage decision keeps options open.

Dantzig's legacy: stochastic programming in scenario trees

- Take a deterministic time-staged linear program.
- Build a scenario tree with recourse at different stages.
- Solve a big optimization problem.
- Extend this methodology to many new applications that have uncertainty...

But often "not to tree" is better

- Uncertainties in long-term planning models (e.g. PILOT)
  - technological change;
  - politics;
  - demand;
  - probabilities difficult to estimate from historical data;
  - a scenario tree can help guide first stage decision.
- Uncertainties in shorter time frames (e.g. cycling a battery/solar system, hydro scheduling)
  - stochastic processes can be estimated from historical data ;
  - independence/simple time dependence in random variables;
  - low decision dimension enables dynamic programming (stochastic control) ;
  - solution is a policy (decision rule), which does not need a tree.

Summary

Summary

Deterministic social plan

Consider a single node electricity system with generators $i \in \mathcal{G}$ meeting demand d. A system operator solves

$$\begin{aligned} \text{S: min} \quad & \textstyle\sum_{i \in \mathcal{G}} c_i(q_i) \\ & \textstyle\sum_{i \in \mathcal{G}} q_i = d, \quad [\pi] \\ & q_i \in Q_i. \end{aligned}$$

Suppose for each generator i, $c_i(q_i)$ is a strictly convex function of q_i and Q_i is a nonempty convex set.

Lagrangian

$$\min_{q_i \in Q_i} \quad \sum_{i \in \mathcal{G}} c_i(q_i) + \pi(d - \sum_{i \in \mathcal{G}} q_i)$$

If each generator i solves

$$q_i^*(\pi) = \arg\max_{q_i \in Q_i} \{\pi q_i - c_i(q_i)\}$$

and $\sum_{i \in \mathcal{G}} q_i^*(\pi) = d$ then $q_i^*(\pi)$ solves social plan S.

System operator: out-of-market dispatch versus central dispatch

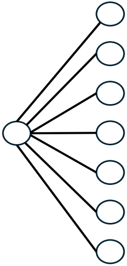
- If the system operator has full information then it can announce a price π and knows that each generator $i \in G$ will generate $q_i^*(\pi)$ when optimizing its profit.
- If the system operator does not have full information (e.g. a new solar farm might be available without the system operator's knowledge, which reduces net demand) then it might announce the wrong price, and dispatch might not be system optimal.
- Central dispatch requires each generator to provide a decision rule to the system operator, namely "if the price is π, then I will generate $q_i^*(\pi)$".
- Decision rules in convex market setting are (non-decreasing) supply functions.

Supply functions give "wait-and-see" decisions

- When demand is random, a supply function gives a wait-and-see decision.
- If only demand varies over time, long-lived supply functions are optimal for many trading periods.
- What about unit-commitment, ramping, etc.?
- Two-stage optimization yields stochastic dispatch models.

Summary

Scenario tree model
[Pritchard et al, (2010)]



Scenario tree of 7 outcomes $\omega \in \Omega$.

Pre-commitment and real time

- q_i is a day-ahead setpoint level for generator i, with convex cost $c_i(q_i)$.
- d is consumer demand d accruing welfare $w(d)$.
- $X_i(\omega)$ is the real-time dispatch produced by generator i in scenario $\omega \in \Omega$.
- $U_i(\omega)$ is the amount by which generator i deviates from q_i in scenario ω.
- $r_i(U_i(\omega))$ is the (convex) cost incurred by generator i for its deviation.
- $G_i(\omega)$ the maximum output capacity of generator i in scenario ω.
- $\xi(\omega)$ random renewable generation (wind) in scenario ω.

Stochastic dispatch

$$
\begin{array}{llll}
\text{SD:} & \min & \sum_i c_i\left(q_i\right) - w(d) + \sum_i \sum_\omega \mathbb{P}(\omega) r_i\left(U_i(\omega)\right) & \\
 & \text{s.t.} & \sum_i X_i(\omega) \geq d - \xi(\omega), & [\mathbb{P}(\omega)\pi(\omega)] \\
 & & q + U(\omega) = X(\omega), & \\
 & & 0 \leq X(\omega) \leq G(\omega), \quad q \geq 0. &
\end{array}
$$

Payment mechanism using stochastic prices

- System operator solves SD.
- Each scenario yields a random price $\pi(\omega)$.
- If scenario ω eventuates then
  - demand pays $d\pi(\omega)$
  - generator is paid $X(\omega)\pi(\omega)$
  - wind is paid $\xi(\omega)\pi(\omega)$
  - we call this the stochastic prices payment mechanism.

Theorem

For every ω

$$d\pi(\omega) = X(\omega)\pi(\omega) + \xi(\omega)\pi(\omega)$$

- The amount or revenue collected from demand is enough to cover the payment to generators.

Revenue adequacy and cost recovery

[Cory Wright et al. (2018)]

Definition

A payment mechanism for SD is revenue adequate if and only if in every scenario $\omega \in \Omega$, the amount or revenue collected from demand is enough to cover the payment to generators. (Also called budget balanced.)

Definition

A payment mechanism for SD exhibits cost recovery if and only if, in every scenario $\omega \in \Omega$, all generators recover their short-run (fuel and deviation) costs. (Also called individually rational).

Revenue adequacy and cost recovery
[Cory Wright et al. (2018)]

Theorem

Under the stochastic prices payment scheme, each agent i recovers expected costs.

Theorem

There is no payment scheme that makes a solution to SD budget balanced and individually rational in each scenario.

Coherent risk measures reminder
[Artzner, Delbaen, Eber, Heath, (1999)]

A coherent risk measure is a mapping ρ from a space $\mathbb{Z}$ of random variables to $\mathbb{R}$ that satisfies the following axioms for Z_1 and $Z_2 \in \mathbb{Z}$.

Subadditivity: $\rho(Z_1 + Z_2) \leq \rho(Z_1) + \rho(Z_2)$;

Monotonicity: If $Z_1 \leq Z_2$, then $\rho(Z_1) \leq \rho(Z_2)$;

Positive homogeneity: If $c \in \mathbb{R}$ and $c \geq 0$, then

$$\rho(cZ_1) = c\rho(Z_1);$$

Translation equivariance: If $c \in \mathbb{R}$, then

$$\rho(c + Z_1) = c + \rho(Z_1).$$

Dual representation

A coherent risk measure of a random disbenefit Z can be expressed as

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $\mathcal{D}$ is a convex set of probability measures called the risk set.

Risk-averse agents

We assume each generator $i \in \mathcal{G}$ (and demand $i = 0$) is risk averse with a coherent risk measure ρ_i. If $Z_i(\omega)$ is the random disbenefit of agent i then

$$\rho_i(Z) = \max_{\mathbb{Q} \in \mathcal{D}^i} \mathbb{E}_{\mathbb{Q}}[Z_i(\omega)]$$

where generator $i \in \mathcal{G}$ has convex risk set $\mathcal{D}^i$ and demand has convex risk set $\mathcal{D}^0$.

Complete market for risk

Definition

An Arrow-Debreu security for scenario ω is a financial instrument that pays \$1 if ω occurs and \$0 otherwise.

- We assume that there is an Arrow-Debreu security for each ω that can be traded in an exchange.
- The price at which Arrow Debreu security ω sells for is $\mu(\omega)$.
- The market clears at price $\mu(\omega)$.

$$0 \leq -\sum_{i\in\mathcal{G}} W_i(\omega) - W_0(\omega) \perp \mu(\omega) \geq 0$$

- The payoff for i is

$$A_i(W_i) = -\sum_{\omega} \mu(\omega) W_i(\omega) + W_i(\omega).$$

Risk averse model

Let

$$\mathcal{D} = \cap_{i \in \mathcal{G}} \mathcal{D}^i \cap \mathcal{D}^0$$

$$\begin{array}{llll} \text{RASD:} & \min & \max\limits_{\mathbb{Q} \in \mathcal{D}} & \sum_\omega \mathbb{Q}(\omega) \sum_i \left(c_i\left(q_i\right) - w(d) + r_i\left(U_i(\omega)\right)\right) \\ & & \text{s.t.} & \sum_i X_i(\omega) \geq d - \xi(\omega), \qquad [\pi(\omega)] \\ & & & q + U(\omega) = X(\omega), \\ & & & 0 \leq X(\omega) \leq G(\omega), \quad q \geq 0. \end{array}$$

Risk-adjusted pricing

[Leclere & P. (2025)]

Theorem

RASD yields locational marginal energy prices that are revenue adequate in every scenario, and, after trading Arrow-Debreu securities yielding random returns, each generator agent i optimizes profit in risk-adjusted expectation.

$$\mathcal{G}: \min_{q_i, U_i, W_i} \max_{\mathbb{Q} \in \mathcal{D}^i} \sum_{\omega} \mathbb{Q}(\omega) \left[c_i(q_i) + r_i(U_i(\omega)) - \pi(\omega) q_i - A_i(W_i) \right] \leq 0$$

$$\text{Demand: } \min_{d, W_0} \max_{\mathbb{Q} \in \mathcal{D}^0} \sum_{\omega} \mathbb{Q}(\omega) [d\pi(\omega) - A_0(W_0)] - w(d) \leq 0$$

(Note: Wind makes non-negative profit in every scenario)

Summary so far

- Risk-adjusted pricing gives budget balance and risk-adjusted individual rationality.
- Assumes truthful representation of risk measure to system operator (incentive compatibility)...

...but risk-adjusted pricing is not incentive compatible. [Myerson & Satterthwaite (1983), Leclere & P. (2025)].

- Assumes all agents share the same (finite) probability distribution of W.
- Assumes a convex, two-stage model for dispatch.
- Assumes a complete market for risk trading.

Next part of tutorial...

- What happens with many periods,. . .
- . . . or longer time scales?
- Is a tree then the best model for future uncertainty?
- How to allow markets to establish a "wisdom of crowds".

References

[1] Artzner, P., Delbaen, F., Eber, J.M. and Heath, D., 1999. Coherent measures of risk. Mathematical finance, 9(3), pp.203-228.

[2] Cory-Wright, R., Philpott, A. and Zakeri, G., 2018. Payment mechanisms for electricity markets with uncertain supply. Operations Research Letters, 46(1), pp.116-121.

[3] Ferris, M. and Philpott, A., 2022. Dynamic risked equilibrium. Operations Research, 70(3), pp.1933-1952.

[4] Leclère, V. and Philpott, A., 2025. Strategic behavior of risk-averse agents under stochastic market clearing, www.epoc.org.nz.

[5] Myerson, R.B. and Satterthwaite, M.A., 1983. Efficient mechanisms for bilateral trading. Journal of Economic Theory, 29(2), pp.265-281.

References

[6] Pritchard, G., Zakeri, G. and Philpott, A., 2010. A single-settlement, energy-only electric power market for unpredictable and intermittent participants. Operations Research, 58(4-part-2), pp.1210-1219.
[7] Ralph, D. and Smeers, Y., 2015. Risk trading and endogenous probabilities in investment equilibria. SIAM Journal on Optimization, 25(4), pp.2589-2611.
[8] Roald, L.A., Pozo, D., Papavasiliou, A., Molzahn, D.K., Kazempour, J. and Conejo, A., 2023. Power systems optimization under uncertainty: A review of methods and applications. Electric Power Systems Research, 214, p.108725.

Incentive compatibility example
[Leclere & P. (2025)]

- Risk-averse consumer with true risk set $\mathcal{D}$ has decreasing marginal utility for energy in period 2 (demand curve);
- Risk averse generator with true risk set $\mathcal{D}$ sets gas plant output x in period 1 for supply in period 2;
- In period 2 in scenario ω, generator supplies extra dispatch $U(\omega)$ and sees random wind output $W(\omega)$;
- Under truthful declaration, system operator optimizes total welfare using risk set $\mathcal{D}$ that dispatches high x being averse to low $W(\omega)$.
- But generator prefers low x since this makes prices higher in period 2. It therefore declares itself to be risk neutral so system operator will dispatch more x.