Multistage and medium-term models

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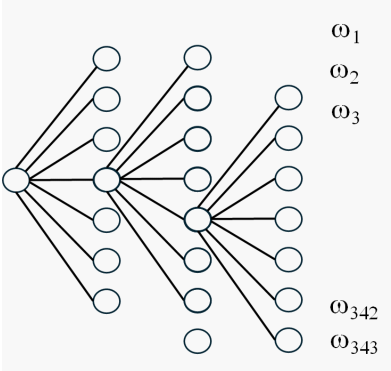
This tutorial in three parts

1. Short-term models (hours/days)
    - social plan minimizing cost
    - maximizing profit given prices
2. Multistage and medium-term models (weeks/months)
    - social plan minimizing cost
    - maximizing profit given prices
3. Long-term models (years/decades)
    - social plan minimizing cost
    - maximizing profit given prices

Summary

Summary

Multi-stage scenario tree



Multistage scenario tree with three stages. Each rank of nodes corresponds to a different time stage.

Multi-stage equilibrium theory

[Ferris and P. (2022)]

- Considers a scenario tree of nodes $n \in \mathcal{N}$.
- A multistage risk-averse social planning problem RASP is formulated in $\mathcal{N}$.
  - assumes a nested coherent risk measure with one-step risk sets $\mathcal{D}(n)$ in the interior of positive orthant;
  - planner minimizes risk-adjusted expected cost of meeting demand;
- Solution to RASP defines dual variables $\pi(n)$ at each node n.

Multi-stage equilibrium theory

- Assume agent i maximizes risk-adjusted expected profit using price process $\pi(n), n \in \mathcal{N}$.
  - uses a nested coherent risk measure with one-step polyhedral risk sets $\mathcal{D}_i(n)$ in the interior of positive orthant;
  - can trade risk in each node n using a complete market of Arrow-Debreu securities;
  - at each node $n \in \mathcal{N}$, $\cap_i \mathcal{D}_i(n) \neq \emptyset$.

Theorem

A. If $\mathcal{D}(n) = \cap_i \mathcal{D}_i(n)$ then the prices $\pi(n)$, $n \in \mathcal{N}$, and actions from RASP define a Walrasian equilibrium where each agent maximizes risk-adjusted expected profit.
B. Suppose the prices $\pi(n)$, $n \in \mathcal{N}$, give optimal risk adjusted actions for each agent that forms an equilibrium. Then these actions solve RASP where $\mathcal{D}(n) = \cap_i \mathcal{D}_i(n)$.

Drawbacks of this in practice

- The equilibrium involves each agent maximizing a multistage risk-averse problem with a tree of prices.
    - This becomes intractable very quickly.
    - Stochastic process of prices is poorly behaved even if demand is stagewise independent [Barty et al. (2010)]
    - Truncate to a few stages and solve in rolling horizon mode?
- The equilibrium involves each agent using the same tree as the system operator.
- Markets should aggregate different beliefs of agents (putting "money where their mouths are").

Rolling horizon optimization

- Deterministic multistage rolling horizon (model predictive control) used in many electricity pools.
- Useful for modeling intertemporal constraints
  - river-chain linkages
  - storage e.g. charging/discharging batteries
  - ramping constraints
  - unit commitment
- Can lead to "inconsistent prices".

Inconsistent prices

	generation		battery	capacity =	4						
	capacity	7		initial charge =	4				Solve 1	Solve 2	Solve 3
time	cost	generation	discharge	charge	battery	supply	demand		Price	Price	Price
1	8	6	0	0	4	6	6		8		
2	5	7	0	0	4	7	7		8	5	
3	12	4	4	0	0	8	8		12	12	12
	cost	131	battery	revenue =	48						

Inconsistent prices

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2	5	7	0	4	4	3	7		8	5	
3	12	4	4	0	0	8	8		12	12	12
	cost	131	battery	revenue =	60						

Summary

Stochastic optimal control

Many multistage stochastic programs in energy are convex stochastic optimal control problems.

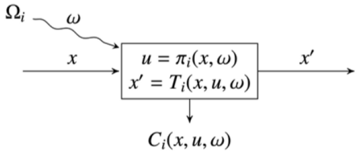
$$\begin{array}{rl} \text{SOC: min} & \mathbb{F}[\sum_{t=1}^{T} c(t)u(t)] \\ \text{s.t.} & x(t+1) = x(t) + B(t)u(t) + \xi(t), \\ & u(t) \in U(x, t, \xi(t)), \\ & x(t) \in X(t), \end{array}$$

Here $x(t)$ is a state vector, $u(t)$ the control, $\xi(t)$ is a random noise that is stagewise independent and $\mathbb{F}$ is some convex risk functional, typically $\mathbb{E}$.

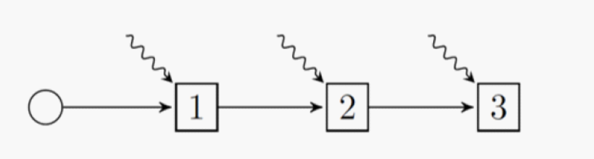
Stochastic dual dynamic programming

- In many cases SOC can be solved using dynamic programming. Avoids the need for a scenario tree.
- Most popular approach in energy is Stochastic Dual Dynamic Programming(SDDP) [Pereira & Pinto (1991)].
- SDDP has been widely applied in hydrothermal scheduling.
- Julia package `SDDP.jl` [Dowson & Kapelevich (2021)]
  - JuMP model defines the stage problem.
  - policy graph defines the dynamics and noise.
- Output is an approximately optimal policy defined by cutting planes.
- `SDDP.jl` Julia Library downloadable from `https://odow.github.io/SDDP.jl/stable/`

Policy graphs in SDDP.jl



Typical stage problem in SDDP



Policy graph for three stage stochastic program

JADE.jl

`JADE.jl` is a hydrothermal reservoir optimization model of the New Zealand electricity system that minimizes expected discounted social cost. It applies the stochastic dual dynamic programming algorithm as implemented in the Julia package `SDDP.jl` developed by Oscar Dowson.

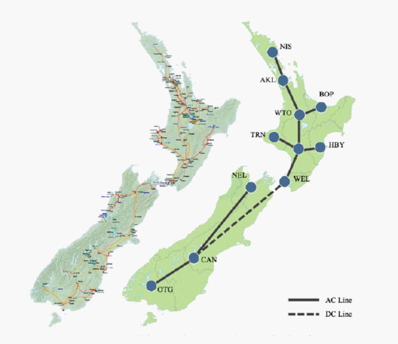
`JADE.jl` is made available by Electricity Authority at

`https://www.emi.ea.govt.nz/Wholesale/Tools/JADE`

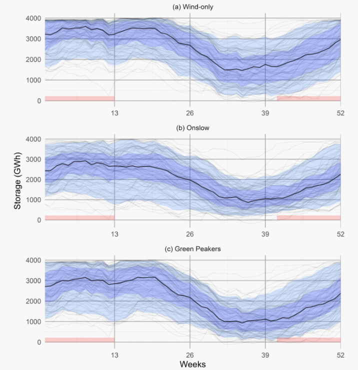
and source is on github at

`https://github.com/EPOC-NZ/JADE.jl`

JADE.jl



Transmission network for JADE model. Majority of hydro storage in South Island.



Simulation of national reservoir storage for optimal steady-state policy under three investment scenarios.

Case study: Shut down coal generation
[Fulton (2018), Ferris & P.(2023)]

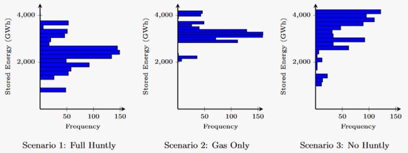


Figure 8: Start of Year NZ Storage.

End of year national storage for three coal shutdown scenarios.

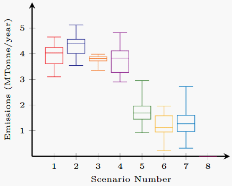


Figure 9: Distribution of Carbon Emissions by Scenario

Distribution of carbon emissions for coal shutdown scenarios.

Summary

Dynamic programming and market equilibrium

- Ferris-Philpott model establishes dynamic Walrasian equilibrium using prices.[Ferris & P. (2022)]
- Generator optimization using price process requires a scenario tree.
- Seek equilibrium in policies.

Definition

A Recursive Competitive Equilibrium (RCE) is a state-dependent decision rule for each agent, and a state-dependent price process in which each agent maximizes their reward at prices and supply equals demand in each state and stage.

- RCE developed for macroeconomic models. [Huggett (1993), Aiyagari (1995), Pagnocelli et al (2025)]

Recursive Competitive Equilibrium maximizes welfare
[Philpott et al (2024)]

- Solve social planning model using dynamic programming (SDDP).
- Yields optimal actions and prices in a scenario tree.
- Optimal actions and prices and actions form a Walrasian dynamic equilibrium.
- Assumption: social Bellman function the sum of of agent Bellman functions.
- Each convex stage problem decouples by agent into a profit maximization at clearing prices.
- Social planning solution coincides with RCE.

Example: Battery optimization

- One generator, one battery
- State is previous period dispatch $\bar{x}$ and battery charge $\bar{y}$
- Actions are dispatch x, discharge u and charge v
- Ramp limits

$$\mathcal{X}(\bar{x}) = \{x \mid 0 \leq x \leq q, x - \bar{x} \leq \rho, \bar{x} - x \leq \sigma\}$$

- Dynamics of battery charging

$$\mathcal{Y}(\bar{y}) = \{(y, u, v) \mid 0 \leq y \leq E, 0 \leq u \leq r, 0 \leq v \leq s, y = \bar{y} - u + \eta v\}$$

Example: Battery optimization

Stage problem for SDDP is

$$
\begin{array}{rll}
F^{t-1}(x(t-1), y(t-1)) = & \min & c^t(x) + Lz + F^t(x,y) \\
& \text{s.t.} & x + u - v + z = d(t) + w, \\
& & x \in \mathcal{X}(x(t-1)), \\
& & (y,u,v) \in \mathcal{Y}(y(t-1)), \\
& & w \geq 0, z \in [0, d(t)],
\end{array}
$$

where $F^{24}(x,y) = 0$ and $x(0) = x^0$, $y(0) = y^0$.

Example: Parameter values

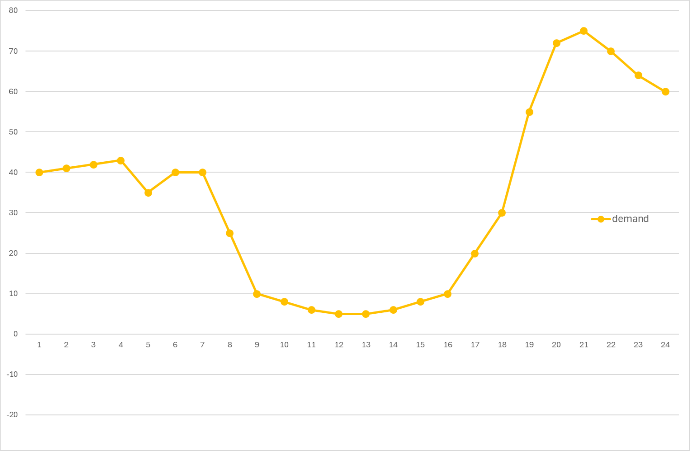
energy tranche	5	5	5	5	5	5	10	10	10	10
marginal cost	10	20	30	40	50	70	90	110	150	200

Marginal cost of generator ($/MWh) increasing in 10 steps.

$q = 70.0$	$E = 30.0$	$\eta = 1.0$
$r = 15.0$	$s = 15.0$	$\rho = 10.0$
$L = 1000.0$	$x^0 = 35.0$	$y^0 = 0.0$

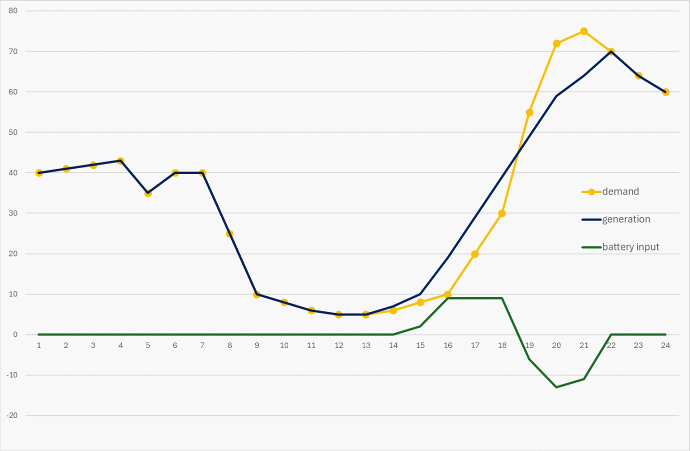
Table: Parameter values for example

Example: Net demand forecast



Net demand for $t = 1, 2, \ldots, 24$.

Example: Socially optimal solution with perfect foresight



Optimal dispatch x (blue), net charge $v - u$ (green) for $t = 1, 2, \ldots, 24$.Minimum cost = \$48,470.

Example of agent decision rules

- Perfect foresight solution has objective function value \$48,470.
- Add noise $\{-4, -2, 0, 2, 4\}$ at each t to the demand, and apply SDP: optimal expected cost of \$52,377.
- SDP policy (cost-to-go function) is not the sum of agent cost-to-go functions, so RCE not socially optimal.
- Try simulating three different agent policies (10000 sample paths).
  - Perfect foresight policy: \$54,255 (s.e. \$57).
  - Separable approximation to perfect foresight policy: \$55,430 (s.e. \$76).
  - Separable approximation to SDP policy: \$52,688 (s.e. \$55).

Summary

Challenges and opportunities

- RCE requires common view of the future uncertainty, but dispatch works with decision rule even if not a RCE.
- Do decision rules work in real dispatch settings (e.g., with unit commitment)?
- Strategic behaviour of storgage (Markov Perfect Equilibrium).
- Understanding aggregating different agent views of the future (wisdom of crowds).

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